MATH 255 DIFFERENTIAL EQUATIONS
Final Exam / 07.01.2025

Name: Number:

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1	2	3	4	5	6	Total
20	20	15	20	20	25	120

SOLUTION KEY

1) (a) (7 points) (WebWork) *Find the solution to the initial value problem*

$$\frac{y' - e^{-t} + 6}{y} = -6, \quad y(0) = 5.$$

SOLUTION: $y' - e^{-t} + 6 = -6y \Rightarrow y' + 6y = e^{-t} - 6 \Rightarrow y' \cdot e^{6t} + 6e^{6t}y = (e^{-t} - 6)e^{6t}$

$$\Rightarrow (ye^{6t})' = e^{5t} - 6e^{6t} \Rightarrow y(t)e^{6t} - y(0) = \int_0^t (e^{5s} - 6e^{6s})\,ds$$

$$\Rightarrow y(t)e^{6t} - 5 = \frac{e^{5t}}{5} - e^{6t} - \frac{1}{5} + 1$$

$$\Rightarrow \boxed{y(t) = \frac{29}{5}e^{-6t} + \frac{e^{-t}}{5} - 1}$$

For the following items of the question put a check sign √ if it is true or put a × if it is wrong.

(b) (3 points) (WebWork) *Which of the following differential equations are exact?*

(i) ☑ $(5y - 2x)y' - 2y = 0.$

(ii) ☑ $(2y\sin(x)\cos(x) - y + 2y^2e^{xy^2})dx = (x - \sin^2(x) - 4xye^{xy^2})dy.$

(iii) ☒ $\left(1 - \frac{3}{y} + x\right)\frac{dy}{dx} = \frac{3}{x} - 1.$

SOLUTION: (i) $\left.\begin{array}{l} M(x,y) = -2y \Rightarrow M_y(x,y) = -2. \\ N(x,y) = 5y - 2x \Rightarrow N_x(x,y) = -2 \end{array}\right\} M_y = N_x.$

(ii) $M(x,y) = y\sin(2x) - y + 2y^2e^{xy^2} \Rightarrow M_y(x,y) = \sin(2x) - 1 + 4ye^{xy^2} + 4xy^3e^{xy^2}$

$N(x,y) = -x + \sin^2(x) + 4xye^{xy^2} \Rightarrow N_x(x,y) = -1 + 2\cdot\sin(x)\cdot\cos(x) + 4ye^{xy^2} + 4xy^3e^{xy^2}$

(iii) $\left.\begin{array}{l} M(x,y) = \frac{3}{x} - 1 \Rightarrow M_y(x,y) = 0 \\ N(x,y) = 1 - 3/y + x \Rightarrow N_x(x,y) = 1 \end{array}\right\} M_y \neq N_x.$

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(c) (3 points) (WebWork) *Which of the following differential equations are separable?*

(i) ✓ $\frac{dy}{dt} = ty - y$

(ii) ✓ $\frac{dy}{dt} = \frac{\sin(t)\cos(y)}{1+y^2}$

(iii) ✗ $\frac{dy}{dt} = t^2 - y^2$

SOLUTION:

(i) $\frac{dy}{dt} = y.(t-1) \Rightarrow \frac{dy}{y} = (t-1)dt$

(ii) $\frac{1+y^2}{\cos(y)} dy = \sin(t)dt$

(d) (3 points) (WebWork) *Mark the correct one and fill in the blanks.*

(i) $x\frac{dy}{dx} - 4y = x^6e^x$ *is a linear / nonlinear differential equation of order* $\boxed{1}$.

(ii) $\left(\frac{dy}{dx}\right)^2 + y\cos(x) = 5$ *is a linear / nonlinear differential equation of order* $\boxed{1}$.

(iii) $\frac{d^2y}{dx^2} + \sin(x)\frac{dy}{dx} = \cos(x)$ *is a linear / nonlinear differential equation of order* $\boxed{2}$.

***(e) (4 points)** Match each linear system with the one of the phase plane direction fields.*

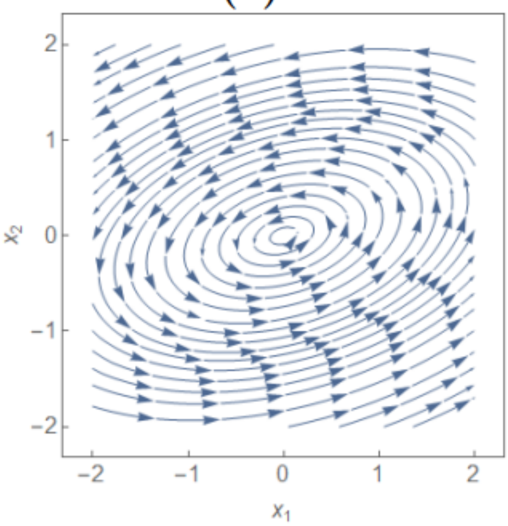
A. $\boxed{1}$ $$\begin{aligned} x_1{}' &= x_1 - 5x_2 \\ x_2{}' &= 2x_1 - x_2 \end{aligned}$$

B. $\boxed{2}$ $$\begin{aligned} x_1{}' &= x_1 - 4x_2 \\ x_2{}' &= 4x_1 - 7x_2 \end{aligned}$$

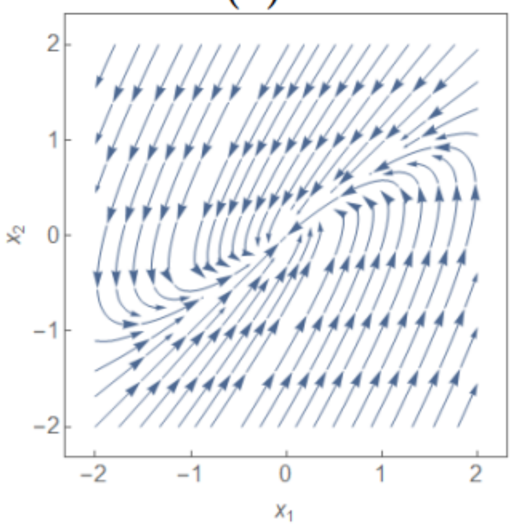
C. $\boxed{3}$ $$\begin{aligned} x_1{}' &= x_1 + 4x_2 \\ x_2{}' &= x_1 - 2x_2 \end{aligned}$$

D. $\boxed{4}$ $$\begin{aligned} x_1{}' &= 3x_1 + 4x_2 \\ x_2{}' &= -2x_1 - x_2 \end{aligned}$$

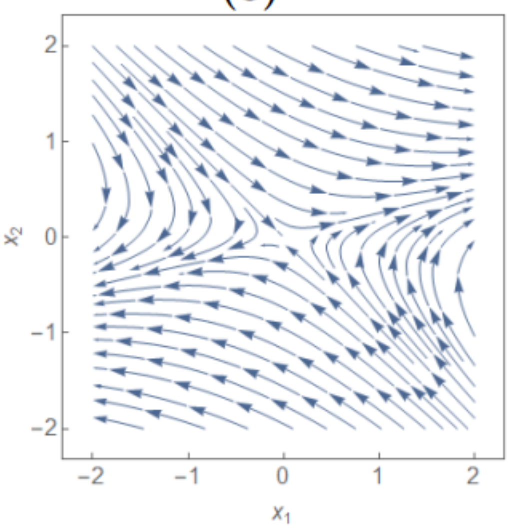
(1)



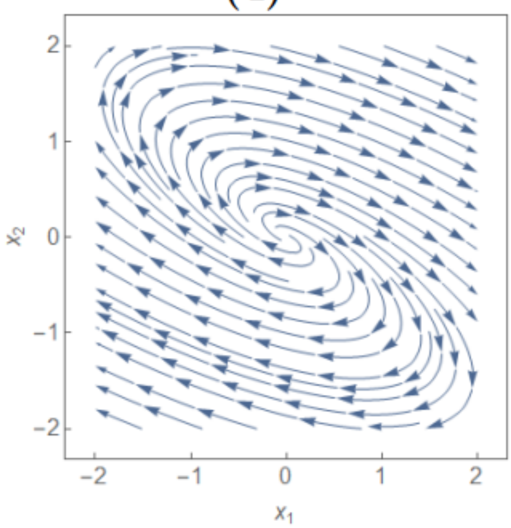
(2)



(3)



(4)



2) *(a)* **(9 points)** *Consider the differential equation*

$$y'' - 6y' + \gamma y = 2e^{2x} + e^{3x}(x - \sin(x))\,.$$

For $\gamma = 8, \gamma = 9$, and $\gamma = 10$, find the final form for the associated particular solution (taking into account the linear dependencies) if the method of undetermined coefficients is to be applied (do not evaluate the coefficients).

(i) $\gamma = 8$: The roots of characteristic equation $r^2 - 6r + 8 = 0$ are $r_1 = 4$ and $r_2 = 2$. Solutions of homogeneous part are $y_1(x) = e^{4x}$ and $y_2(x) = e^{2x}$.

$$\boxed{y_p(x) = c_1 \cdot x \cdot e^{2x} + e^{3x} \cdot (c_2 x + c_3) + e^{3x} \cdot (c_4 \cdot \cos(x) + c_5 \cdot \sin(x))}$$

(ii) $\gamma = 9$: The roots of characteristic equation $r^2 - 6r + 9 = 0$ are $r_1 = r_2 = 3$. Solutions of homogeneous part are $y_1(x) = e^{3x}$ and $y_2(x) = x e^{3x}$.

$$\boxed{y_p(x) = c_1 e^{2x} + c_2 x^2 e^{3x} + e^{3x}(c_3 \cdot \cos(x) + c_4 \cdot \sin(x))}$$

(iii) $\gamma = 10$: The roots of characteristic equation $r^2 - 6r + 10 = 0$ are $r_1 = 3 + i$ and $r_2 = 3 - i$. Solutions of homogeneous part are $y_1(x) = e^{3x} \cdot \cos(x)$ and $y_2(x) = e^{3x} \cdot \sin(x)$.

$$\boxed{y_p(x) = c_1 e^{2x} + e^{3x} \cdot (c_2 x + c_3) + e^{3x}(c_4 x \cos(x) + c_5 x \sin(x))}$$

b) (11 points) *Given that* $y_1(x) = xe^{-x}$ *and* $y_2(x) = xe^{-2x}\sin(x)$ *are two fundamental solutions to a sixth-order, constant coefficient, linear, homogeneous differential equation. Find the other fundamental solutions and express the general solution. Explain your reasoning.*

SOLUTION : Since we have x in front of e^{-x}. It means that there are repeated roots. So $y_3(x) = e^{-x}$ is also a fundamental solution.

Similarly, $y_4(x) = e^{-2x}\sin(x)$, $y_5(x) = e^{-2x}\cos(x)$ and $y_6(x) = x e^{-2x}\cos(2x)$ are fundamental solutions.

The general solution is

$$y(x) = (c_1 + c_2 x)e^{-x} + e^{-2x}\left((c_3 + c_4 x).\cos(x) + (c_5 + c_6 x)\sin(x)\right)$$

3) ***(15 points)*** *Find the general solution to the differential equation*

$$x^2y'' + 6xy' + 6y = 2x^{-1}e^{-x}, \qquad x > 0.$$

SOLUTION: Let $y_h(x) = x^r$. Then $y_h'(x) = r.x^{r-1}$ and

$y_h''(x) = r.(r-1).x^{r-2}$.

$$x^2.(r.(r-1).x^{r-2}) + 6x.(r.x^{r-1}) + 6.x^r = 0$$

$$r.(r-1).x^r + 6rx^r + 6x^r = 0 \Rightarrow (r^2 - r + 6r + 6)x^r = 0$$

$$\Rightarrow r^2 + 5r + 6 = 0 \quad \Rightarrow (r+2).(r+3) = 0$$

$$\Rightarrow r_1 = -2 \quad r_2 = -3.$$

So $y_h(x) = c_1.x^{-2} + c_2.x^{-3}$.

We set $y_p(x) = x^{-2}u_1(x) + x^{-3}u_2(x)$ where

$$x^{-2}u_1'(x) + x^{-3}u_2'(x) = 0$$

$$-2x^{-3}u_1'(x) - 3x^{-4}u_2(x) = \frac{2x^{-1}e^{-x}}{x^2} = 2x^{-3}e^{-x}$$

Now,

$$W = \begin{vmatrix} x^{-2} & x^{-3} \\ -2x^{-3} & -3x^{-4} \end{vmatrix} = -3x^{-6} + 2x^{-6} = -x^{-6},$$

$$W_1 = \begin{vmatrix} 0 & x^{-3} \\ 2x^{-3}e^{-x} & -3x^{-4} \end{vmatrix} = -2x^{-6}e^{-x}$$

$$W_2 = \begin{vmatrix} x^{-2} & 0 \\ -2x^{-3} & 2x^{-3}e^{-x} \end{vmatrix} = 2x^{-5}e^{-x}$$

$$u_1' = \frac{W_1}{W} = \frac{-2x^{-6}e^{-x}}{-x^{-6}} = 2e^{-x} \quad \text{and}$$

$$u_2' = \frac{W_2}{W} = \frac{2x^{-5}e^{-x}}{-x^{-6}} = -2xe^{-x}$$

Integrating yields

$$u_1 = -2e^{-x} \quad \text{and} \quad u_2 = 2xe^{-x} + 2e^{-x}.$$

Therefore;

$$y_p(x) = x^{-2}(-2e^{-x}) + x^{-3}(2xe^{-x} + 2e^{-x})$$

$$y_p(x) = \cancel{-2x^{-2}e^{-x}} + \cancel{2x^{-2}e^{-x}} + 2x^{-3}e^{-x}$$

So the general solution of differential equation is

$$\boxed{y(x) = c_1 \cdot x^{-2} + c_2 \cdot x^{-3} + 2x^{-3}e^{-x}}$$

4) *(a)* (**15 points**) *Solve the following system of first-order equations*

$$y' = \begin{bmatrix} 11 & -25 \\ 4 & -9 \end{bmatrix} y.$$

(**b**) (**5 points**) *Find a fundamental matrix.*

SOLUTIONS: (a) The characteristic equation of the given system of first-order equation is

$$\begin{vmatrix} 11-\lambda & -25 \\ 4 & -9-\lambda \end{vmatrix} = 0 \quad \Rightarrow \quad (\lambda-1)^2 = 0.$$

Hence the roots of the equation are $\lambda_1 = \lambda_2 = 1$.

Then, we will find eigenvector corresponding to eigenvalue $\lambda_1 = \lambda_2 = 1$.

For $\lambda = 1$,

$$\begin{pmatrix} 10 & -25 \\ 4 & -10 \end{pmatrix} \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \end{pmatrix} \quad \Rightarrow \quad 10v_{11} - 25v_{12} = 0$$

$\Rightarrow 10v_{11} = 25v_{12} \Rightarrow v_{11} = \frac{25}{10} v_{12}$. Then the eigenvector is

$$v_1 = \begin{pmatrix} v_{11} \\ v_{12} \end{pmatrix} = \begin{pmatrix} \frac{25}{10} v_{12} \\ v_{12} \end{pmatrix} = \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} v_{12}.$$

We will find a generalized eigenvector of rank 2 corresponding to eigenvalue $\lambda = 1$.

If v_2 is a generalized eigenvector of rank 2 corresponding to eigenvalue $\lambda = 1$

$$(A - I)^2 v_2 = 0 \text{ , where } A = \begin{pmatrix} 11 & -25 \\ 4 & -9 \end{pmatrix}.$$

$$(A - I) \cdot (A - I) v_2 = 0 .$$

We have known that $(A - I) v_1 = 0$, so we have

$$(A - I) v_2 = v_1 .$$

$$\begin{pmatrix} 10 & -25 \\ 4 & -10 \end{pmatrix} \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} = \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} \Rightarrow 4 v_{21} - 10 v_{22} = 1$$

$$\Rightarrow v_{21} = \frac{1 + 10 v_{22}}{4} = \frac{1}{4} + \frac{5}{2} v_{22}$$

$$v_2 = \begin{pmatrix} v_{21} \\ v_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} + \frac{5}{2} v_{22} \\ v_{22} \end{pmatrix} = \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix} + \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} v_{22}$$

Finally, the general solution of given system of differential equations is

$$y(t) = c_1 \cdot e^t \cdot \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} + c_2 \cdot \left(e^t \cdot t \begin{pmatrix} \frac{5}{2} \\ 1 \end{pmatrix} + e^t \begin{pmatrix} \frac{1}{4} \\ 0 \end{pmatrix} \right)$$

$$\boxed{y(t) = (c_1 + c_2 t) e^t \cdot \begin{pmatrix} 5/2 \\ 1 \end{pmatrix} + c_2 \cdot e^t \cdot \begin{pmatrix} 1/4 \\ 0 \end{pmatrix}}$$

(b) A fundamental matrix is

$$\boxed{\begin{pmatrix} \frac{5}{2} e^t & \frac{5}{2} \cdot t \cdot e^t + \frac{1}{4} e^t \\ e^t & t \cdot e^t \end{pmatrix}}$$

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5) **(WebWork)** *Find the inverse Laplace transform* $f(t) = \mathcal{L}^{-1}\{F(s)\}$ *of the functions*

(i) **(10 points)** $F(s) = \frac{8s-29}{s^2-8s+20}$.

(ii) **(10 points)** $F(s) = \frac{6}{s^2-5s+4}\, e^{-5s}$. *(Notation: Use unit step functions* $u_c(t)$ *with step at* $t = c$ *to represent* $f(t)$*).*

SOLUTIONS:

(i) $$F(s) = \frac{8s-29}{s^2-8s+20} = \frac{8s-29}{(s^2-8s+16)+4} = \frac{8s-29}{(s-4)^2+2^2}$$

$$= \frac{8.(s-4)+3}{(s-4)^2+2^2} = 8.\frac{s-4}{(s-4)^2+2^2} + \frac{3}{2}.\frac{2}{(s-4)^2+2^2}$$

$$f(t) = \mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{8.\frac{s-4}{(s-4)^2+2^2} + \frac{3}{2}.\frac{2}{(s-4)^2+2^2}\right\}$$

$$= 8.e^{4t}.\cos(2t) + \frac{3}{2}.e^{4t}.\sin(2t).$$

(ii) $$\mathcal{L}^{-1}\{F(s)\} = \mathcal{L}^{-1}\left\{e^{-5s}\frac{6}{s^2-5s+4}\right\} = u_5(t)\, f(t-5)$$

where $f(t) = \mathcal{L}^{-1}\left\{\frac{6}{s^2-5s+4}\right\}$.

$$\mathcal{L}^{-1}\left\{\frac{6}{s^2-5s+4}\right\} = \mathcal{L}^{-1}\left\{2.\frac{1}{s-4} + (-2).\frac{1}{s-1}\right\} = 2.e^{4t} - 2.e^{t}$$

$$\mathcal{L}^{-1}\{F(s)\} = u_5(t).\left(2.e^{4t-20} - 2.e^{t-5}\right).$$

6) (WebWork) *Find the solution to the given initial value problem using Laplace transformation*

$$\frac{d^2y}{dt^2} + 9y = g(t), \quad y(0) = 0,\ y'(0) = 0$$

where $g(t) = \begin{cases} t, & 0 \le t < 4 \\ 0, & 4 \le t < \infty \end{cases}$.

(i) **(5 points)** *Take the Laplace transform of both sides of the given differential equation to create the corresponding algebraic equation. Denote the Laplace transform of y(t) by Y(s). Do not move any terms from one side of the equation to the other (until you get to part (2) below).*

____________________ = ____________________

(ii) **(5 points)** *Solve your equation for Y(s).*

$Y(s) = \mathcal{L}\{y(t)\} =$ ____________________

(iii) **(15 points)** *Take the inverse Laplace transform of both sides of the previous equation to solve for y(t).*

If necessary, use unit step functions to represent your solution.

$y(t) =$ ____________________

Write your explanations on finding $y(t)$ related with item (iii) below in detail.

SOLUTIONS (i) Taking the Laplace transform of both of sides of the given differential equation

$$\mathcal{L}\{y'' + 9y\} = \mathcal{L}\{g(t\}$$

$$\mathcal{L}\{y''\} + 9 \cdot \mathcal{L}\{y\} = \mathcal{L}\{g(t)\}$$

$$s^2 Y(s) - sy(0) - y'(0) + 9 \cdot Y(s) = G(s),$$

where $G(s) = \mathcal{L}\{g(t)\} = \mathcal{L}\{t \cdot [u(t) - u(t-4)]\}$

$$= \mathcal{L}\{u_0(t) \cdot t\} - \mathcal{L}\{u_4(t) \cdot t\}$$

$$= e^{-0s} F_1(s) - e^{-4s} F_2(s)$$

$$F_1(s) = \mathcal{L}\{f_1(t)\} = \mathcal{L}\{t\} = \frac{1}{s^2},$$

$$F_2(s) = \mathcal{L}\{f_2(t)\} = \mathcal{L}\{t+4\} = \frac{1}{s^2} + \frac{4}{s}.$$

Since $y(0) = y'(0) = 0$, we can obtain

$$\boxed{s^2 Y(s) + 9Y(s) = \frac{1}{s^2} - e^{-4s} \cdot \left(\frac{1}{s^2} + \frac{4}{s}\right)}$$

(ii) $$Y(s) \cdot (s^2+9) = \frac{1}{s^2} - e^{-4s} \cdot \left(\frac{1}{s^2} + \frac{4}{s}\right)$$

$$\boxed{Y(s) = \frac{1}{s^2+9} \cdot \left[\frac{1}{s^2} - e^{-4s} \cdot \left(\frac{1}{s^2} + \frac{4}{s}\right)\right].}$$

(iii) We will take the inverse Laplace transform of both of sides of previous equation to solve $y(t)$.

$$y(t) = \mathcal{L}^{-1}\{Y(s)\} = \mathcal{L}^{-1}\left\{\frac{1}{s^2+9} \cdot \left[\frac{1}{s^2} - e^{-4s} \cdot \left(\frac{1}{s^2} + \frac{4}{s}\right)\right]\right\}$$

$$= \mathcal{L}^{-1}\left\{\frac{1}{s^2 \cdot (s^2+9)}\right\} - \mathcal{L}^{-1}\left\{e^{-4s} \cdot \frac{1}{s^2 \cdot (s^2+9)}\right\}$$

$$- \mathcal{L}^{-1}\left\{\frac{4e^{-4s}}{s \cdot (s^2+9)}\right\}.$$

Since

$$\frac{1}{s^2 \cdot (s^2+9)} = \frac{1}{9s^2} - \frac{1}{9(s^2+9)} \quad \text{and} \quad \frac{1}{s(s^2+9)} = \frac{1}{9s} - \frac{s}{9(s^2+9)}$$

$$\mathcal{L}^{-1}\left\{\frac{1}{s^2 \cdot (s^2+9)}\right\} = \frac{t}{9} - \frac{\sin(3t)}{27}$$

$$\mathcal{L}^{-1}\left\{e^{-4s} \cdot \frac{1}{s^2 \cdot (s^2+9)}\right\} = u_4(t) \cdot \left(\frac{t-4}{9} - \frac{\sin(3t-12)}{27}\right)$$

$$\mathcal{L}^{-1}\left\{\frac{4e^{-4s}}{s \cdot (s^2+9)}\right\} = u_4(t)\left(\frac{4}{9} - \frac{4\cos(3t-12)}{9}\right).$$

Then, the solution to the initial value problem is

$$\boxed{\begin{aligned} y(t) = {} & \frac{t}{9} - \frac{\sin(3t)}{27} \\ & - u_4(t)\left(\frac{t-4}{9} - \frac{\sin(3t-12)}{27}\right) \\ & - u_4(t)\left(\frac{4}{9} - \frac{4\cos(3t-12)}{9}\right). \end{aligned}}$$

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Table of Laplace Transforms

	$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$		$f(t)=\mathfrak{L}^{-1}\{F(s)\}$	$F(s)=\mathfrak{L}\{f(t)\}$
1.	1	$\frac{1}{s}$	2.	$\mathbf{e}^{at}$	$\frac{1}{s-a}$
3.	$t^n,\quad n=1,2,3,\ldots$	$\frac{n!}{s^{n+1}}$	4.	$t^p, p>-1$	$\frac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sqrt{t}$	$\frac{\sqrt{\pi}}{2s^{\frac{3}{2}}}$	6.	$t^{n-\frac{1}{2}},\quad n=1,2,3,\ldots$	$\frac{1\cdot3\cdot5\cdots(2n-1)\sqrt{\pi}}{2^n s^{n+\frac{1}{2}}}$
7.	$\sin(at)$	$\frac{a}{s^2+a^2}$	8.	$\cos(at)$	$\frac{s}{s^2+a^2}$
9.	$t\sin(at)$	$\frac{2as}{(s^2+a^2)^2}$	10.	$t\cos(at)$	$\frac{s^2-a^2}{(s^2+a^2)^2}$
11.	$\sin(at)-at\cos(at)$	$\frac{2a^3}{(s^2+a^2)^2}$	12.	$\sin(at)+at\cos(at)$	$\frac{2as^2}{(s^2+a^2)^2}$
13.	$\cos(at)-at\sin(at)$	$\frac{s(s^2-a^2)}{(s^2+a^2)^2}$	14.	$\cos(at)+at\sin(at)$	$\frac{s(s^2+3a^2)}{(s^2+a^2)^2}$
15.	$\sin(at+b)$	$\frac{s\sin(b)+a\cos(b)}{s^2+a^2}$	16.	$\cos(at+b)$	$\frac{s\cos(b)-a\sin(b)}{s^2+a^2}$
17.	$\sinh(at)$	$\frac{a}{s^2-a^2}$	18.	$\cosh(at)$	$\frac{s}{s^2-a^2}$
19.	$\mathbf{e}^{at}\sin(bt)$	$\frac{b}{(s-a)^2+b^2}$	20.	$\mathbf{e}^{at}\cos(bt)$	$\frac{s-a}{(s-a)^2+b^2}$
21.	$\mathbf{e}^{at}\sinh(bt)$	$\frac{b}{(s-a)^2-b^2}$	22.	$\mathbf{e}^{at}\cosh(bt)$	$\frac{s-a}{(s-a)^2-b^2}$
23.	$t^n\mathbf{e}^{at},\quad n=1,2,3,\ldots$	$\frac{n!}{(s-a)^{n+1}}$	24.	$f(ct)$	$\frac{1}{c}F\left(\frac{s}{c}\right)$
25.	$u_c(t)=u(t-c)$ Heaviside Function	$\frac{\mathbf{e}^{-cs}}{s}$	26.	$\delta(t-c)$ Dirac Delta Function	$\mathbf{e}^{-cs}$
27.	$u_c(t)f(t-c)$	$\mathbf{e}^{-cs}F(s)$	28.	$u_c(t)g(t)$	$\mathbf{e}^{-cs}\mathfrak{L}\{g(t+c)\}$
29.	$\mathbf{e}^{ct}f(t)$	$F(s-c)$	30.	$t^n f(t),\quad n=1,2,3,\ldots$	$(-1)^n F^{(n)}(s)$
31.	$\frac{1}{t}f(t)$	$\int_s^\infty F(u)\,du$	32.	$\int_0^t f(v)\,dv$	$\frac{F(s)}{s}$
33.	$\int_0^t f(t-\tau)g(\tau)\,d\tau$	$F(s)G(s)$	34.	$f(t+T)=f(t)$	$\frac{\int_0^T \mathbf{e}^{-st}f(t)\,dt}{1-\mathbf{e}^{-sT}}$
35.	$f'(t)$	$sF(s)-f(0)$	36.	$f''(t)$	$s^2F(s)-sf(0)-f'(0)$
37.	$f^{(n)}(t)$	$s^nF(s)-s^{n-1}f(0)-s^{n-2}f'(0)\cdots-sf^{(n-2)}(0)-f^{(n-1)}(0)$			

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