11.03.2025

İzmir Institute of Technology
MSE 222 Applied Mathematics for Materials Science and Engineering, Spring 2025
Quiz I – Solution Key

Name: ______________________________

Student ID: ______________________________

Duration: 50 Minutes

Grade Table

Question:	1	2	3	4	5	6	Total
Points:	16	16	24	8	12	24	100
Score:							

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1. (16 points) If

$$A = \begin{bmatrix} -1 & 2 & 4 \\ 4 & -3 & 1 \\ 0 & -3 & -4 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 3 & -3 \\ -4 & 4 & -4 \\ -2 & 3 & 3 \end{bmatrix}$$

then,

$$2A - 3B = \begin{bmatrix} -8 & -5 & 17 \\ 20 & -18 & 14 \\ 6 & -15 & -17 \end{bmatrix} \text{ and } A^T = \begin{bmatrix} -1 & 4 & 0 \\ 2 & -3 & -3 \\ 4 & 1 & -4 \end{bmatrix}.$$

2. (16 points) If A and B are 7×4 matrices, and C is a 9×7 matrix, which of the followings are defined? Use checkmark ($\checkmark$) if it is, crossmark ($\times$) otherwise.

(a) $B + A$ $\boxed{\checkmark}$

(b) $A^T C^T$ $\boxed{\checkmark}$

(c) BA $\boxed{\times}$

(d) CA $\boxed{\checkmark}$

3. Consider the following system of equations

$$-3x + 6y = -12,$$
$$7x - 13y = 23.$$

(a) (18 points) Solve the system by applying the steps below:

Step 1: The initial augmented form for the linear system is

$$\left[\begin{array}{cc|c} -3 & 6 & -12 \\ 7 & -13 & 23 \end{array}\right].$$

Step 2: First, perform the row operation $-\frac{1}{3}R_1 \to R_1$. The resulting matrix is

$$\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 7 & -13 & 23 \end{array}\right].$$

Step 3: Next, perform the row operation $-7R_1 + R_2 \to R_2$. The resulting matrix is

$$\left[\begin{array}{cc|c} 1 & -2 & 4 \\ 0 & 1 & -5 \end{array}\right].$$

Step 4: Finish simplifying the augmented matrix to reduced row echelon form. The reduced matrix is

$$\left[\begin{array}{cc|c} 1 & 0 & -6 \\ 0 & 1 & -5 \end{array}\right].$$

We performed $2R_2 + R_1 \to R_1$ to end up with reduced row echelon form.

(b) (6 points) What are the solutions to the system? If there are no solutions, write "no solution". If there are infinitely many solutions, let $y = t$ and solve x in terms of t. If there is a unique solution, write x and y.

The coefficients matrix is row equivalent to I_2, therefore there is a unique solution. $x = -6$, $y = -5$.

4. The reduced row echelon forms of the augmented matrices of two systems are given below. Mark the correct one for each of the followings.

(a) (4 points)

$$\left[\begin{array}{ccc|c} 0 & 1 & 0 & 15 \\ 0 & 0 & 1 & 16 \end{array}\right].$$

(A) No solutions

(B) $\boxed{\text{Infinitely many solutions}}$

(C) Unique solution

(D) None of the above

(b) (4 points)

$$\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$$

(A) Unique solution: $x = 0$, $y = 0$, $z = 0$

(B) Infinitely many solutions

(C) Unique solution: $x = 1$, $y = 1$, $z = 0$

(D) No solutions

(E) $\boxed{\text{Unique solution: } x = 0,\ y = 0}$

F None of the above

5. Let

$$A = \begin{bmatrix} 1 & 0 & -3 & 0 & 2 \\ 0 & 1 & 4 & 0 & 5 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

(a) (4 points) Is the matrix in row echelon form? [$\boxed{\text{Yes}}$ / ~~No~~]

(b) (4 points) Is the matrix in reduced row echelon form? [$\boxed{\text{Yes}}$ / ~~No~~]

(c) (4 points) If this matrix were the augmented matrix for a system of linear equations, would the system be inconsistent or consistent? [~~Inconsistent~~ / $\boxed{\text{Consistent}}$]

6. Consider the following Gauss-Jordan reduction:

$$\underbrace{\begin{bmatrix} 0 & 7 & 0 \\ 1 & 49 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{A} \xrightarrow{-7R_1+R_2\to R_2} \underbrace{\begin{bmatrix} 0 & 7 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_1A} \xrightarrow{R_1\leftrightarrow R_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 7 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_2E_1A} \xrightarrow{(1/7)R_2\to R_2} \underbrace{\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}}_{E_3E_2E_1A}.$$

(a) (12 points) Find

$$E_1 = \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_2 = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}, \quad E_3 = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{7} & 0 \\ 0 & 0 & 1 \end{bmatrix},$$

(b) (12 points) Find A^{-1}. (Hint: Observe that according to the Gauss-Jordan reduction, A is row equivalent to I_3.)
From the above Gauss-Jordan process, we have

$$E_3E_2E_1A = I_3.$$

Therefore, $A^{-1} = E_3E_2E_1$:

$$A^{-1} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \frac{1}{7} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ -7 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -7 & 1 & 0 \\ \frac{1}{7} & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}.$$