MSE 222
Applied Mathematics for Material Science and Engineering
2025–2026, Spring Term

Course Syllabus

Instructor	Dr. Kemal Cem Yılmaz	**Time and Venue**	on Mon., 13.30–15.15, class Z-41
E-mail	cemyilmaz@iyte.edu.tr		on Wed., 13.30–15.15, class Z-41
Address	Office 116, Dept. of Mathematics		at Eng. Faculty Building D

Course Description

This is a one semester course with 4 credit and 7 ECTS (according to the undergraduate curriculum 2024). The course consists of two main parts; Part I: Linear Algebra and Part II: Partial Differential Equations.

Part I: Linear Algebra. Linear systems of equations arise in various applications of engineering, and linear algebra provides the mathematical framework for understanding and analyzing these systems. It makes systematic use of vectors and matrices to represent them, which naturally leads to the study properties of matrices and relevant topics.

To this end, Part I covers the fundamental concepts of linear algebra including matrix arithmetic, determinants, systems of linear equations, vector spaces, notions of dimension and spanning set, finite-dimensional inner product spaces, linear transformations, eigenvalues and eigenvectors.

Part II: Partial Differential Equations (PDEs). Differential equations are mathematical models that describe several physical phenomena from micro world to macro world. It is a mathematical relation that involves instantaneous rates of change –so called derivatives– of a state with respect to independent variable(s). In many physical situations, in order to obtain a more realistic model, it is necessary to account for both spatial and temporal rates of change. Such equations serve as mathematical models describing the evolution of physical phenomena in a spatial domain, therefore referred to as evolutionary PDEs. They are important as they arise in connection with various physical phenomena in engineering and science that vary both in time and in space.

In Part II of this course, the first goal is introduce an inner product for functions in the space of piecewise continuous functions and the notion of orthogonality in this setting. We introduce a special set of trigonometric functions that forms an orthonormal (Fourier) basis for this space, leading naturally to an infinite-dimensional inner product space. Next, we cover Fourier series expansion of periodic functions, as well as the even and odd extensions of piecewise continuous functions defined on a finite interval. The remaining part of the course focuses on modeling of the continuity equation, shows how different choices of vector fields lead to different evolutionary PDEs, and, as a canonical example, demonstrates the use of Fourier series to solve the one-dimensional heat equation.

Learning Outcomes

Upon successfully completing this course, it is expected that students have following outcomes:

1. Perform elementary matrix operations.
2. Express linear systems of equations in matrix form.
3. Perform row operations. Be able to solve linear systems of equations via Gauss elimination and Gauss-Jordan elimination methods.
4. Compute determinants and solve linear systems of equations via Cramer's rule.

5. Determine whether a solution to a given linear systems of equations is certain to exist, is certain to unique.
6. Determine whether a given set of vectors are linear dependent or independent.
7. Define and determine the notions of dimension, basis, spanning set of a vector space, rank of a matrix.
8. State and explain the inner product space axioms.
9. Understand the notions of orthogonality, orthogonal basis. Express a vector of $\mathbb{R}^n$ as a linear combination of a given orthogonal basis.
10. Use the Gram-Schmidt orthogonalization process to construct an orthogonal set of vectors.
11. Express the matrix representation of a linear transformation on $\mathbb{R}^n \to \mathbb{R}^m$ with respect to the standard basis.
12. Compute eigenvalues and corresponding eigenvectors of a linear transformation on $\mathbb{R}^n$. Interpret eigenpairs geometrically.
13. Explain the notion of orthogonality for functions belonging to the space of piecewise continuous functions
14. Represent periodic functions and functions defined on a finite interval by their Fourier series.
15. Explain the principle of superposition for linear PDEs.
16. Distinguish between initial and boundary value problems. Interpret the physical meaning of Dirichlet and Neumann boundary conditions.
17. Explain the underlying geometric and physical principles used in modeling the continuity equation.
18. Solve the one-dimensional heat (and wave) equations posed on a finite interval using separation of variables.

Prerequisites

No prerequisite is required for Part I: Linear Algebra. Regarding Part II: Partial Differential Equations, you are expected to be familiar with the following topics:

- Even and odd functions (Math 141)
- Integration techniques for functions of single variables (Math 141 or Math 145)
- Notion of series (Math 142)
- Techniques for solving constant coefficient linear first-order and second-order ordinary differential equations (Math 255)
- Vector fields, flux integral, gradient, divergence and curl operators. (Math 241)

Course Materials

Communication. All announcements will be posted via Microsoft Teams. Please use the code

`untwghk`

to enroll our Team Room "*MSE222: Applied Mathematics for MSE, Spring 25-26*" in Microsoft Teams.

Teaching. A graphic tablet together with a projector will be used for teaching in the classes. The lecture notes will be saved and shared via Microsoft Teams on the same day.

We may utilize from GeoGebra™ and MATLAB™ to present some simulations during the classes. GeoGebra is free. We have a campus wide license for MATLAB. You can visit https://bidb.iyte.edu.tr/matlab/ to have the product on your device.

Textbooks. The following textbooks will serve as the primary course resources.

[L] Steven J. Leon, *Linear Algebra with Applications*, 8th ed., Pearson, 2010.

[H] Richard Haberman, *Elementary Applied Partial Differential Equations with Fourier Series and Boundary Value Problems*, 3rd ed., Prentice-Hall Inc., 1998.

[K] Erwin Kreyszig, *Advanced Engineering Mathematics*, 10th ed., John Wiley and Sons Inc, 2011.

Please see the Course Outline part below for the detailed content of the course and the relevant textbook chapters to be covered.

Electronic sources. You can use the following set of video lectures that will aid you to improve your learning on linear algebra and Fourier series.

- Linear Algebra - Video Lectures (1)
- Linear Algebra - Video Lectures (2)
- Fourier Series - Video Lectures

Grading Policy and Expectations

Homework. Each week you will be given a set of exercises via WebWork. The assignments will be based on the topics covered in that week's lectures. Each homework will be announced on Wednesday at 16:00 via Microsoft Teams. Answers to each assignment will be available on the following Saturday at 21:00. Solving these exercises and submitting your works are not mandatory, but it is strongly encouraged. In this way, you are encouraged to maintain a regular weekly study schedule rather than studying just for exams

Quizzes and Exams. You will have 2 Quizzes 10% of your overall grade each, 2 Midterm Examinations 20% of your overall grade each and 1 Final Examination %40 of your overall grade. Each exam will be graded out of 120 points. The extra 20 points will consists of homework questions.

Provided that you correctly solve the homework problems that appear in the exams, you can earn up to an additional 20 points.

	Grade	Weight	Weighted Grade
Quiz 1	100	10%	10
Quiz 2	100	10%	10
Midterm 1	100 + 20	20%	24
Midterm 2	100 + 20	20%	24
Final	100 + 20	40%	48
TOTAL			116

Attendance. Attendance to the lectures is not mandatory, but it is strongly suggested. Should you attend the courses as an active participant during the whole semester continuously, your letter grade may be rounded to the upper one (for those whose letter grade is very close to the upper one).

Based on the above criteria, your Total Grade will be evaluated by the following formula:

$$\text{Total Grade} = 10\% \text{ of Q-I} + 10\% \text{ of Q-II} + 30\% \text{ of M-I} + 30\% \text{ of M-II} + \%40 \text{ of F}.$$

Your Letter Grade will be evaluated according to your Total Grade. Unless indicated otherwise, evaluation of the letter grades will based on the catalog system declared in IZTECH Graduate Education Regulations.

Total Grade	Letter Grade
90–100	AA
85–89	BA
80–84	BB
75–79	CB
70–74	CC
65–69	DC
60–64	DD
50–59	FD
0–49	FF

Office Hours

- Tuesday, 13:30-15:00
- Friday, 13:00-15:00

If you are not available during these hours, send me an e-mail at least 1 day before you want to meet, so we can set up a suitable meeting hour both for us.

Important Dates and Calendar

Quiz 1 .... on Monday, 16th of March
Midterm 1 .... on Monday, 30th of March
Quiz 2 .... on Monday 20th of April
Midterm 2 .... on Monday, 4th of May
Final Exam .... TBA

Detailed information for exams and quizzes will be announced via Microsoft Teams. See also Academic Calendar.

February

S	M	T	W	T	F	S
01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28

March

S	M	T	W	T	F	S
01	02	03	04	05	06	07
08	09	10	11	12	13	14
15	16	17	18	19	20	21
22	23	24	25	26	27	28
29	30	31				

April

S	M	T	W	T	F	S
			01	02	03	04
05	06	07	08	09	10	11
12	13	14	15	16	17	18
19	20	21	22	23	24	25
26	27	28	29	30		

May

S	M	T	W	T	F	S
					01	02
03	04	05	06	07	08	09
10	11	12	13	14	15	16
17	18	19	20	21	22	23
24	25	26	27	28	29	30
31						

June

S	M	T	W	T	F	S
	01	02	03	04	05	06
07	08	09	10	11	12	13
14	15	16	17	18	19	20
21	22	23	24	25	26	27
28	29	30				

- Holidays & Spring Break
- Exams and Quizzes
- Class Days
- Final Weeks

Course Outline

Below is a list of topics that will be covered throughout the semester (section numbers may change depending on the edition of the book).

Part I: Linear Algebra

§I.1 Matrix Operations (Sections 1.3 and 1.4 of [L])

Matrices; equality of matrices; scalar multiplication and addition of matrices, their properties; transpose of matrices; dot product of vectors; matrix-vector and matrix-matrix multiplication; properties of matrix multiplication.

§I.2 Linear System of Equations (Sections 1.1, 1.2 and 1.5 of [L])

Expressing linear systems in matrix form, coefficient and augmented matrices; elementary row operations; row-equivalent systems; Gauss elimination and row-echelon form; reduced row-echelon form; Gauss-Jordan elimination; elementary matrices associated with the elementary row operations; the inverse matrix as a product of elementary matrices; computation of inverse by Gauss-Jordan elimination.

§I.4 Vector Spaces and Linear Transformations (Chapter 3 and Sections 4.1, 4.2 of [L])

Vector space axioms, subspace of a vector space; linear dependence and independence; spanning set for a vector space, basis and dimension; rank of a matrix, null space of a matrix, rank-nullity theorem; linear transformations; the image and the kernel; matrix representations of linear transformations on a finite-dimensional space; some specific linear transformations dilations and contractions, reflections, rotations, translations.

§I.5 Determinants, Cramer's Rule for Solving Linear Systems (Chapter 3 of [L])

Geometric interpretation of the determinant of a $2-$dimensional matrix, understanding what a determinant measures and how to formulate this notion; determinant of an $n-$dimensional matrix, minors, cofactors; properties of determinants; Cramer's rule for solving linear system of equations.

§I.6 Eigenvalues and Eigenvectors of a Matrix (Sections 6.1 and 6.2 of [L])

Eigenvalues and eigenvectors of a matrix, their geometric interpretation; characteristic polynomial; geometric and algebraic multiplicity; some applications of eigenvalue problems.

§I.7 Finite-Dimensional Inner Product Spaces (Sections 5.1, 5.5, 5.6 of [L])

Inner product space axioms; Euclidean norm; orthogonality, orthogonal sets and orthogonal basis; Pythagorean theorem; orthogonal projections and orthogonal decomposition theorem; Gram-Schmidt orthogonalization.

Part II: Partial Differential Equations

§II.1 Fourier Series (Section 5.4 of [L] and Sections 3.1, 3.2. 3.3, 3.6 of [H])

An inner product for piecewise continuous functions, orthogonality in the space of piecewise continuous functions; periodic functions, Fourier series expansion of periodic functions, convergence theorem for Fourier series; even and odd functions, half-range expansion of functions defined on a finite interval, their Fourier series expansion, complex form of Fourier series.

§II.2 A Brief Review of Vector Calculus (Sections 8.9, 8.10 and 8.11 of [K])

Vector field; flux integral, gradient of a scalar field; divergence of a vector field, curl of a vector field, their geometric interpretation; Laplacian; Green's theorem in flux form.

§II.3 Basic Concepts of Differential Equations

Ordinary vs. partial differential equation (PDE), evolutionary PDEs; initial value problem, boundary value problem, initial–boundary value problem; Dirichlet problem, Neumann problem, Robin problem, mixed boundary conditions, the principle of superposition for linear differential equations.

§III.4 Modeling the Continuity Equation, the Method of Separation of Variables (Chapter 1 and Sections 2.1, 2.2, 2.3, 2.4 of [H])

Modeling the continuity equation in an arbitrary vector field; advection equation - transport by a velocity field; diffusion equation - flux proportional to the gradient; steady-state flows, Laplace and Poisson equations; solving one-dimensional heat equation (and wave equation) posed on a finite interval via separation of variables and use of Fourier series.